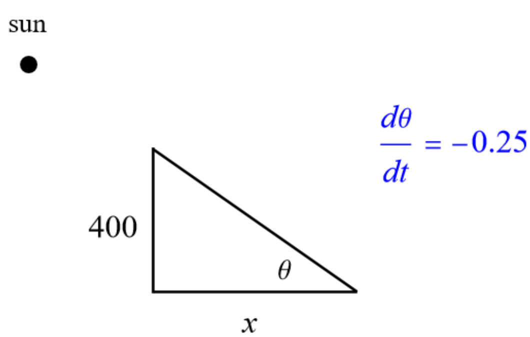
Exercise 101

The angle of elevation of the sun is decreasing at a rate of 0.25 rad/h. How fast is the shadow cast by a 400-ft-tall building increasing when the angle of elevation of the sun is $\pi/6$?

Solution

Draw a schematic of the sun and the building at a certain time.



The aim is to find dx/dt when $\theta = \pi/6$. Take the tangent of the angle to relate θ with the labelled sides of the triangle.

$$\tan\theta = \frac{400}{x} \quad \to \quad x = \frac{400}{\tan\theta}$$

Take the derivative of both sides with respect to time.

$$\frac{d}{dt}(\tan\theta) = \frac{d}{dt}\left(\frac{400}{x}\right)$$

$$(\sec^2\theta)\cdot\frac{d\theta}{dt} = \left(-\frac{400}{x^2}\right)\cdot\frac{dx}{dt}$$

$$\left(\frac{1}{\cos^2\theta}\right)\frac{d\theta}{dt} = -\frac{400}{\left(\frac{400}{\tan\theta}\right)^2}\frac{dx}{dt}$$

$$= -\frac{\tan^2\theta}{400}\frac{dx}{dt}$$

$$= -\frac{\sin^2\theta}{400\cos^2\theta}\frac{dx}{dt}$$

Solve for dx/dt.

$$\frac{dx}{dt} = -\frac{400}{\sin^2\theta}\frac{d\theta}{dt}$$

Therefore, the rate that the shadow is increasing when $\theta = \pi/6$ is

$$\left.\frac{dx}{dt}\right|_{\theta=\pi/6} = -\frac{400}{\sin^2\frac{\pi}{6}}(-0.25) = 400\ \frac{\text{ft}}{\text{hr}}.$$